Black Turkeys, Fat Tails, and a Gaggle of Economists

While the circumstances of this market crash might be unique, are such drops normal for equity markets, or is this time different?

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Key Takeaways

- While market crashes have been called black swans (a black swan being a unique negative event that could not be foreseen because no similar events had occurred in the past), a careful look at historical market data reveals that large market corrections occur on average about every nine years. Larry Siegel, director of research at the CFA Institute Research Foundation, coined a more appropriate term: black turkey. He defines a black turkey as "an event that is everywhere in the data—it happens all the time—but to which one is willfully blind."

- The financial industry tends to rely on lognormal distributions to model returns. Lognormal distributions have thin tails and are based entirely on standard deviation and expected returns. In reality, returns more closely follow log-stable distributions that have fat tails. Log-stable distributions are based on stable Paretian distributions in the same way that lognormal distributions are based on normal distributions. One property of stable Paretian distributions that makes them difficult to work with is that they have an infinite variance. All the foundational models of modern finance, including Markowitz's mean-variance model of portfolio construction, the Capital Asset Pricing Model, and Arbitrage Pricing Theory, depend on the variance of returns being finite. This is one of the main reasons that log-stable distributions have not been widely adopted.

- Stable Paretian models have two shape parameters, peakedness (how high and narrow the center of the distribution is and how fat the tails are) and skewness (which can be negative or positive). Fitting log-stable distributions to monthly returns on the Morningstar equity indexes of 33 countries over the period July 1998 to March 2020, we found that for all countries, the fitted distributions had fat tails. Furthermore, all but one were left skewed.

- Economists have varied and often contradictory explanations of the business cycle. Economists often fall into a few camps including but not limited to the Keynesian, Monetarist, New Classical, Austrian, and Post-Keynesian schools.

- While the coronavirus pandemic may be a once-in-a-lifetime event, its impact on the markets is merely the most recent example of the manifestation of market risk. Unfortunately, this aspect of market risk is not factored into the standard statistical models that we use in finance, despite being consistently present in historical market data.

While the circumstances of the current crash in market value might be unique to the spread of the coronavirus, the more important question is, are such drops normal for equity markets, or is this market crash different?

During the global financial crisis of 2007–09, some observers described the events that were unfolding as a black swan. A black swan is a unique negative event that could not be foreseen because no similar events had occurred in the past (see Taleb 2007).[1] This was a very odd notion to me at the time because I studied long-term capital market data, having worked at Ibbotson Associates, a firm that specialized in collecting historical market returns.[2] I knew from the data that there was a long history of market crashes, some of which were part of a larger financial crisis. Back then, I wrote and co-wrote a series of articles in which I discussed this point in some depth.[3] Five of those articles I collected to form a section of my book on asset allocation (Kaplan 2012).

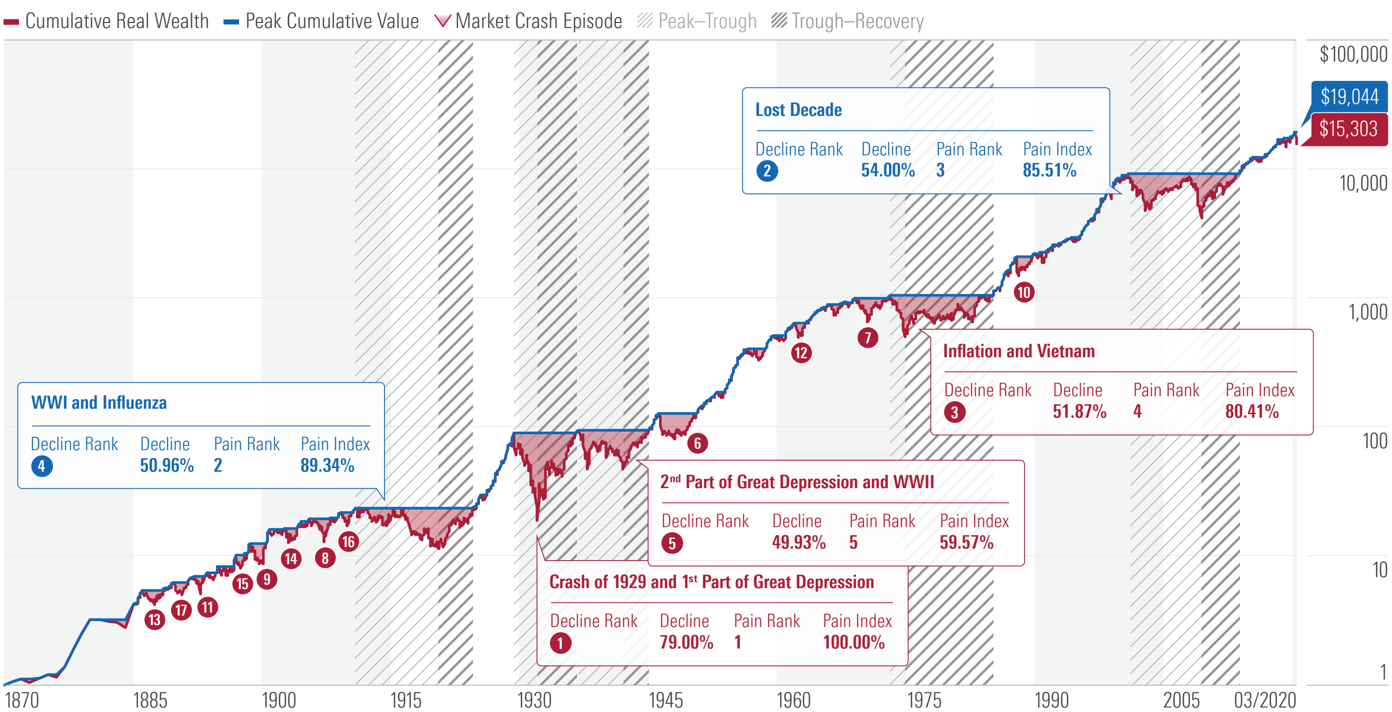
So, what should we call events like the current coronavirus-caused market crisis? Is it a unique black swan event that investors were powerless to see coming? Or is it more like the financial crisis of 2007–09? Larry Siegel, the first employee of Ibbotson Associates and with whom I worked at that firm[4], came up with what I believe is a more fitting term: black turkey. Siegel defines a black turkey as "an event that is everywhere in the data—it happens all the time—but to which one is willfully blind" (Siegel 2010).

So, how many black turkeys have there been? The answer to this question depends on how far back we go in the data and what measure we use to identify black turkeys. Regarding the data, I created a series of real monthly U.S. stock market returns going back to January 1886 with annual returns over the period 1871–85. (I compiled this data for a chapter on the history of market crashes [Kaplan et al. 2009] in a book that Siegel edited [Siegel 2009].) [5]

Exhibit 1 shows the growth of $1 (in 1870 U.S. dollars), with full reinvesting in the stock market index introduced in Kaplan et al. (2009) going back to 1871 and extended until the end of March 2020. It also shows a line that marks the highest market level reached as of each date, the peak value. The gap between the two lines shows declines from the most recent high and the recoveries after the declines. As I discuss below, the area of the gap provides a measure for the severity of the episode of decline. A decline of 20% or more is often considered to be a bear market. Because black turkeys are associated with bear markets, I use bear markets to proxy black turkeys.

1 The black swan is a metaphor for how a single observation can overthrow a long and widely held belief. In Europe, all swans are white, so Europeans believed that all swans everywhere are white. The first sighting by Europeans of a black swan in Australia in 1697 overthrew that belief.
2 Morningstar acquired Ibbotson Associates in 2006.
3 Kaplan et al. (2009), Kaplan (2009a, 2009b, 2009c, 2010), Kaplan, Cooper, Ibbotson, and Mandelbrot (2009).
4 Siegel is now director of research at the CFA Institute Research Foundation. Before that, he was director of investment research at the Ford Foundation.
5 For a description of how these data were compiled from the sources listed under Exhibits 1-3, see Kaplan et al. (2009), Kaplan (2009a), and Ibbotson (2020, Chapter 11).

Exhibit 1 Growth of $1 and Peak Values (Real) of the U.S. Stock Market



Sources: Ibbotson (2020), Morningstar Direct, Goetzmann, Ibbotson, and Peng (2000), Pierce (1982), www.econ.yale.edu/~shiller/data.htm.

Over the period of almost 150 years, Exhibit 1 shows that $1 grows to $15,303. However, there are many drops along the way, some of which were quite severe. Exhibit 2 lists the black turkeys over the 150-year period. There are 17 of them, roughly suggesting that on average they occur about once every nine years (150/17 = 8.8). The worst one was the 79% loss due to the crash of 1929 and the first part of the Great Depression. The second-worst crash was the 54% drop from August 2000 to February 2009. Over this period, the dot-com bubble burst, then there was a recovery, but the recovery was not enough to get the cumulative value back to the August 2000 level (from an inflation-adjusted perspective). Then came the crash of 2007–09. It was not until May 2013, almost 12 and a half years later, that the cumulative value was back at its August 2000 level.

Perhaps the market downturn most relevant to the situation today is fourth-largest decline of almost 51% from June 1911 to December 1920, since this period includes the influenza pandemic of 1918, which was one of the deadliest pandemics in human history.

From Exhibit 2, we see that black turkeys have been occurring throughout the 19th, 20th, and 21st centuries (so far). This is why I labeled this exhibit “A Flock of Turkeys.” Recognizing them in the data should give us a better understanding of risks of equity investing.

In Exhibit 2, I also included a measure of how bad each episode of decline and recovery was—the Pain Index. To measure the Pain Index for each episode, I take the area between the cumulative wealth line and the peak line from peak to recovery. I then divide this by the area for the worst epi-

Exhibit 2 A Flock of Turkeys: The Largest Real Declines in U.S. Stock Market History

Decline Rank	Decline (%)	Peak	Trough	Recovery	Pain Rank	Pain Index (%)	Event(s)
1	79.00	August 1929	May 1932	November 1936	1	100.00	Crash of 1929, 1st Part of Great Depression
2	54.00	August 2000	February 2009	May 2013	3	85.51	Lost Decade: Dot-Com Bubble Burst, Global Financial Crisis
3	51.87	December 1972	September 1974	June 1983	4	80.41	Inflationary Bear Market, Vietnam, Watergate
4	50.96	June 1911	December 1920	December 1924	2	89.34	WWI, Influenza, Postwar Auto Bubble Burst
5	49.93	February 1937	March 1938	February 1945	5	59.57	2nd Part of Great Depression, WWII
6	37.18	May 1946	February 1948	October 1950	6	29.06	Postwar Bear Market
7	35.54	November 1968	June 1970	November 1972	7	14.22	Start of Inflationary Bear Market
8	34.22	January 1906	October 1907	August 1908	8	8.23	Panic of 1907
9	30.41	April 1899	June 1900	March 1901	9	8.18	Cornering of Northern Pacific Stock
10	30.21	August 1987	November 1987	July 1989	10	7.73	Black Monday
11	27.32	October 1892	July 1893	March 1894	16	3.14	Silver Agitation
12	22.80	December 1961	June 1962	April 1963	14	3.55	Cuban Missile Crisis
13	22.04	November 1886	March 1888	May 1889	11	6.25	Depression, Railroad Strikes
14	21.67	April 1903	September 1903	November 1904	12	5.00	Rich Man's Panic
15	21.13	August 1897	March 1898	August 1898	15	3.20	Boer War
16	20.55	September 1909	July 1910	February 1911	17	3.11	Enforcement of the Sherman Anti-Trust Act
17	20.11	May 1890	July 1891	February 1892	13	4.80	Baring Brothers Crisis

Sources: Kaplan et al. (2009); Ibbotson (2020); Morningstar Direct; Goetzmann, Ibbotson, and Peng (2000); Pierce (1982); www.econ.yale.edu/~shiller/data.htm.

sode, which in this case is the August 1929–November 1936 period. The Pain Index gives a perspective on how bad the episodes of decline were, taking into account not only the magnitude of the decline but also how long the decline took to unfold and how long it took to recover.

While I do not have the data to include them, occurrences of black turkeys prior to 1871 have been documented, although, not always in stocks. These include the Dutch Tulip Bulb craze and crash of the of the early 17th century and the South Sea Bubble and crash in early 18th century Britain (Malkiel 2019, Chapter 2).

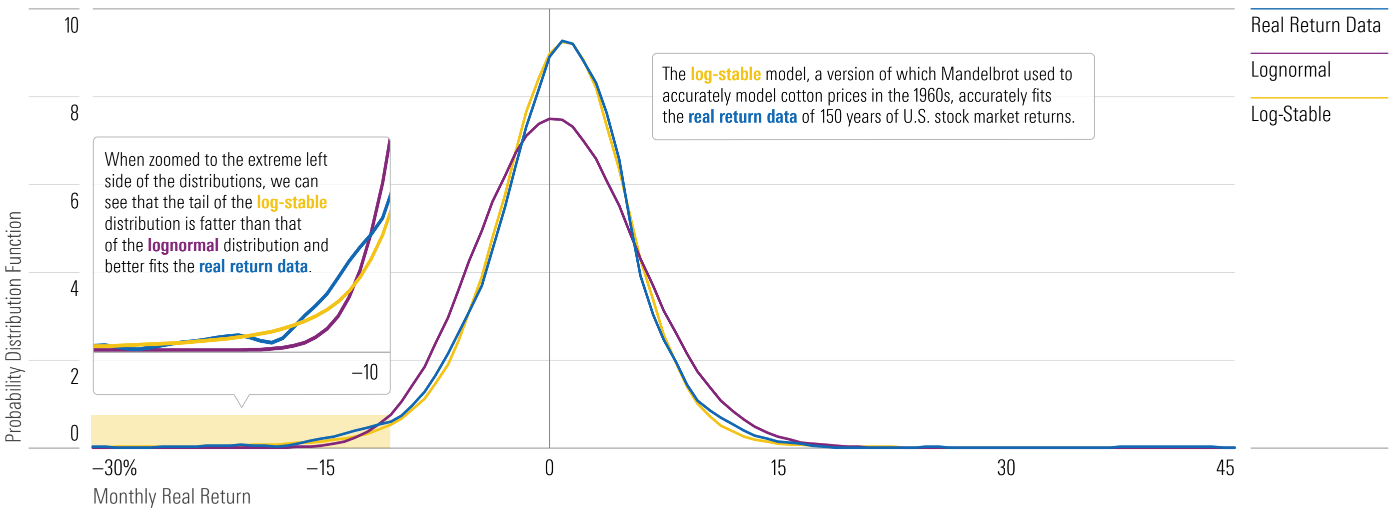
As of March 20, 2020, the decline of the S&P 500 Total Return Index from the high on Feb. 19, 2020, was about 32%, which would rank it at the ninth-worst decline.[6]

Fat Tails

Given the frequency of market declines of 20% or more and the subsequent recoveries, you might think that investment professionals would use models that take the extreme declines during down markets and the rapid rises during recoveries into account when they make investment decisions. But alas, they do not. Rather, investment professionals use tools such as portfolio optimizers and Monte Carlo financial planning models that rely on "thin-tailed models." The ends of a distribution curve are called "tails." They indicate the likelihood of a result on the extreme negative side of the curve or on the extreme positive side. Thin-tail distributions assign probabilities to extreme results that are so

6 The comparison between this marker drop and the ones shown in Exhibit 2 is not exact because I created Exhibit 2 using real (inflation-adjusted) monthly data whereas here I am using nominal daily data.

Exhibit 3 Alternative Distributions for U.S. Monthly Real Stock Market Returns: January 1886–March 2020



Sources: Ibbotson (2020), Morningstar Direct, Goetzmann, Ibbotson, and Peng (2000), Pierce (1982), www.econ.yale.edu/~shiller/data.htm.

low that they can easily be ignored or missed. When you hear market pundits describe a crisis as a “once-in-a-hundred-year event,” they are in effect parroting standard thin-tail models. However, the historical return data tell a vastly different story.

Exhibit 3 shows three types of distribution curves based on the monthly real market returns that I used to create Exhibits 1 and 2: real monthly returns from January 1886 to March 2020. The curve labeled “Lognormal” is the standard distribution model that most of the industry uses. It’s based entirely on the average return and the standard deviation of return, and it resembles a bell curve. This curve has thin tails, meaning that it assigns virtually no probability to extreme returns, either positive or negative. The curve labeled “Real Return Data” is based directly on the historical data and does not rely on any predefined model.[7] Note that the tails of the distribution curve of the actual data are “fatter” than the tails of the lognormal model, both on the left side (which shows extreme negative returns and to the right (extreme positive returns as seen by the length of the right tail with some weight at around 40%).

The curved label “Log-Stable” is an alternative to the standard lognormal model. The log-stable model has fat tails.[8] Notice how well it fits the real return data with a high peak near the middle and fat tails at both ends. In Exhibit 3, we zoomed in on the left side of the distributions to show how much better the log-stable distribution fits the left tail of the data-based distribution.

The log-stable model as a way to model changes in prices was first proposed by the mathematician Benoît Mandelbrot in the 1960s. Mandelbrot is best known for inventing fractal geometry, but back in the 1960s, he took an interest in price changes. He discovered that percentage changes in cotton prices follow a fat-tailed distribution called the stable Paretian distribution. At that time,

7 See Kaplan (2012, Appendix 26A), Kaplan (2018), and Morningstar (2011, Appendix C) for descriptions on how the data-based distribution is created.
8 For detailed descriptions of the log-stable distribution, see Kaplan (2012, Appendix 19A), 2015, and 2019).

he was teaching economics at the University of Chicago and had a doctoral student named Eugene Fama. Fama applied the stable Paretian distribution to changes in the logarithms of stock promises (hence, the log-stable distribution). While the results were good, Fama eventually abandoned this line of research and developed the efficient-market hypothesis, for which he was awarded a Nobel Prize.

If the log-stable model fits the data so well, why did it not become a standard model? One reason could be that a random variable that follows a stable Paretian distribution has infinite variance.[9] The lack of a finite variance means that Markowitz's mean-variance model of portfolio construction (Markowitz 1952, 1959) is invalid. It also means that asset-pricing theories such as the Capital Asset Pricing Model (Sharpe 1964, Lintner 1965) and the Arbitrage Pricing Theory (Ross 1976) are invalid as well. In other words, to accept the log-stable model of returns, we must reject nearly all of standard financial economics.

The uncomfortable implications of returns following log-stable distributions were raised in a very influential book at the time. In 1964, the financial economist Paul H. Cootner published a collection of papers called *The Random Character of Stock Market Prices* (Cootner 1964). This book included a paper by Mandelbrot and a paper by Fama on modeling price changes with stable Paretian distributions. In his introduction to the section containing those papers, Cootner wrote that Mandelbrot forced financial economists "to face up in a substantive way to those uncomfortable empirical observations that there is little doubt most of us have had to sweep under the carpet until now … but surely before consigning centuries of work to the ash pile, we should like to have some assurance that all of our work is truly useless."

As Greg Satell (2010) put it, "In other words, the locomotive was heading down the tracks at full steam, and Mandelbrot would be left at the station. The attractions of financial engineering were too great, the potential profits too gargantuan. While it might be interesting for traders to discuss Mandelbrot's findings over a beer after work, his brand of uncertainty did not win clients nor did it create multi-billion-dollar bonus pools. Sure, there were some problems, but they tweak the models some more and everything would work out in the end. At least they hoped it would. It didn't. The recent financial crises has [sic] laid bare the lie that Mandelbrot exposed more than 40 years ago. As with many of his seemingly outlandish ideas, he had been right all along."

Oh, Canada!

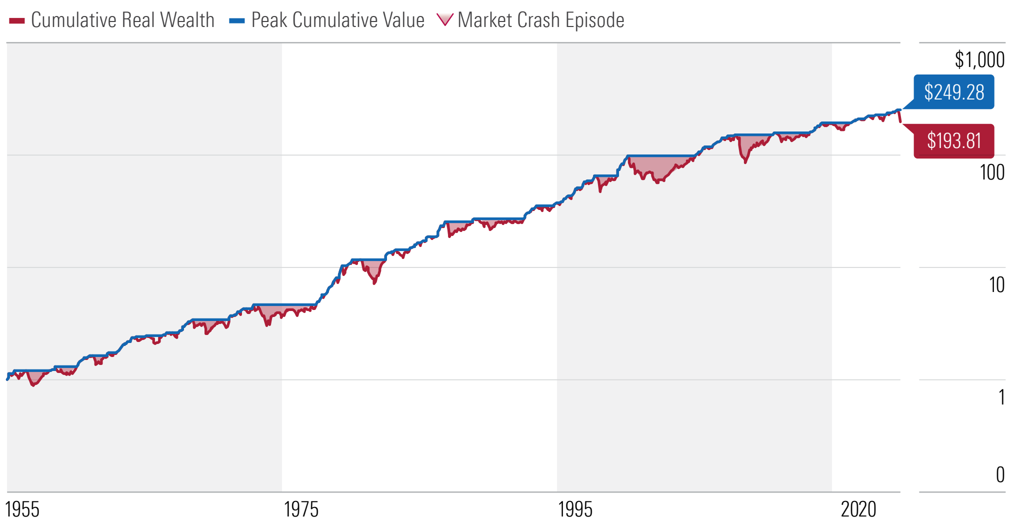
Black turkeys and fat tails are not confined to the United States, but to measure them, we need a long time series of stock market returns. One country for which there is such a series is Canada. A common stock market index for Canada is the S&P/TSX Composite Index, for which monthly total returns are available going back to February 1956. With data going through March 2020, we have just more than 64 years of monthly returns.[10] I obtained this data from Morningstar Direct℠.

9 Morningstar Investment Management does Monte Carlo simulation using a variation on the log-stable distribution called a Truncated Lévy Flight (TLF) that has a finite standard deviation. A TLF distribution is the result of a log-stable distribution that has been truncated on both ends, resulting in a distribution that still has large tails and is skewed, but with a finite variance. See Xiong (2010).

10 Because I did not have monthly inflation rates for Canada going back to February 1956, I used nominal returns.

Exhibit 4 shows the growth of CAD 1.00, with full reinvesting in the stock market index through March 2020, at which point it is worth $193.81. Like Exhibit 1, it also displays a line that shows the highest level received as of each date, so that the gap between the two lines show declines from the most recent high and the recoveries following the declines. As Exhibit 5 shows, ten of these declines were 20% or more. The worst decline was during the global financial crisis of 2007–09. The second-worst one was around the time of dot-com bubble crash. However, as measured by the Pain Index, the dot-com bubble crash was worse, because while the decline almost as same that of the crisis of 2007–09 (around 43%), the period form peak to recovery of the dot-com bubble crush was much longer. Hence, Canada has also had its share of black turkeys.

Exhibit 4 Growth of $1 and Peak Values (CAD) of the S&P/TSX Composite Index



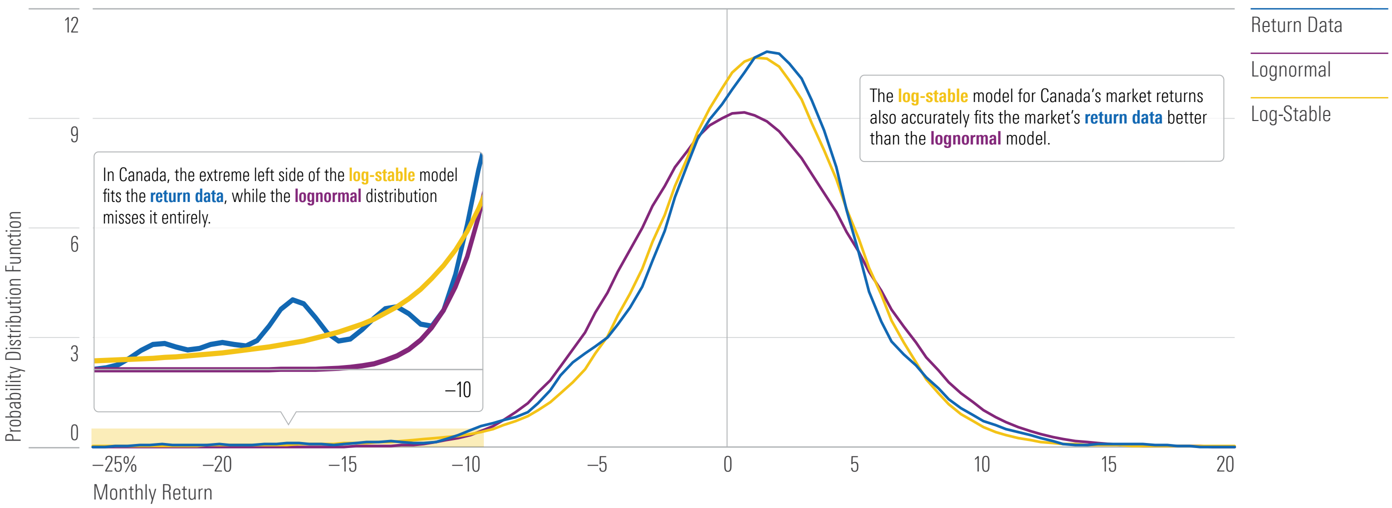
Source: Morningstar Direct.

Exhibit 5 A Flock of Canadian Turkeys: Largest Declines in the S&P/TSX Composite

Decline Rank	Decline (%)	Peak	Trough	Recovery	Pain Rank	Pain Index (%)
1	43.35	May 2008	February 2009	February 2011	3	39.80
2	43.20	August 2000	September 2002	July 2005	1	100.00
3	39.16	June 1981	June 1982	April 1983	4	26.69
4	34.96	October 1973	September 1974	April 1978	2	47.13
5	27.47	April 1998	August 1998	November 1999	9	12.18
6	26.90	May 1957	December 1957	April 1959	7	17.79
7	25.44	July 1987	November 1987	July 1989	8	17.28
8	25.38	May 1969	June 1970	January 1972	5	21.22
9	22.25	January 2020	March 2020	TBD	10	1.72
10	20.08	December 1989	October 1990	March 1993	6	18.48

Source: Morningstar Direct.

Exhibit 6 Alternative Distributions for Monthly Returns (CAD) on the S&P/TSX Composite Index, February 1956–March 2020



Sources: Ibbotson (2020), Morningstar Direct, Goetzmann, Ibbotson, and Peng (2000), Pierce (1982), www.econ.yale.edu/~shiller/data.htm.

Exhibit 6 shows the same three distribution curves in Exhibit 3 but based on the monthly market returns of the S&P/TSX Composite Index from February 1956 to March 2020. As with the U.S., the Return Data curve for Canada clearly has fat tails, both to the left and to the right, but the left tail is especially long. Again, the Lognormal curve, the standard distribution model, has thin tails; it assigns virtually no probability to the extreme returns that did in fact occur. The Log-Stable curve is the Mandelbrot-Fama model. It fits the return data with a high peak near the middle and has fat tails at both ends. The insert shows how well the left tail of the log-stable distribution fits the data, while the lognormal distribution misses it entirely.

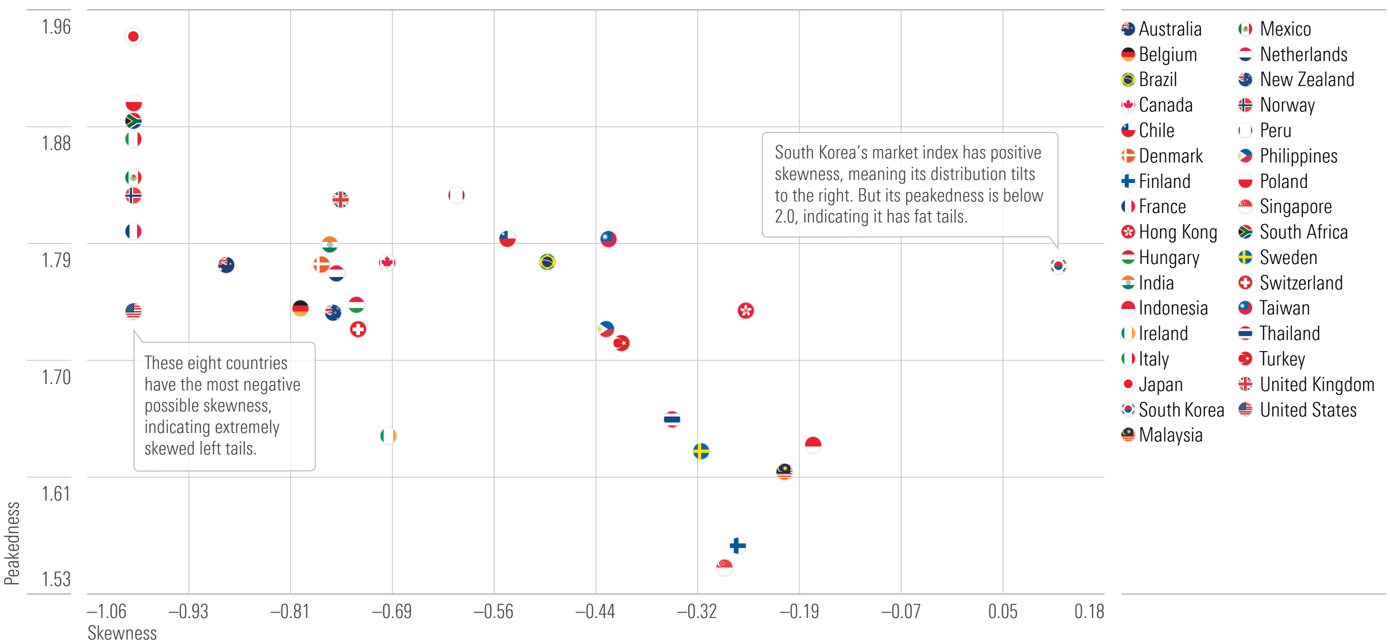
Fat Tails and Skewness Around the World

To further show the benefits of the stable Paretian model and the deficiencies of the lognormal model, we can look at two statistical measures. A stable Paretian distribution has two shape parameters: peakedness and skewness.

The peakedness parameter is between 0 and 2 and indicates how the high point of the distribution curve is much higher than that of a normal or lognormal distribution (as is the case in Exhibits 3 and 6) and has fat tails extending out on both sides. If the peakedness parameter is equal to 2, the distribution is normal with a finite variance. If it is less than 2, the distribution has infinite variance. The more below 2 that the peakedness parameter is, the higher the middle and the fatter the tails.

The skewness parameter indicates the asymmetry of the distribution. The skewness parameter is between negative 1 and positive 1. A value of 0 means that the distribution is symmetric. A negative value means that the distribution is skewed to the left, and a positive value means that it is skewed to the right.

Exhibit 7 Peakedness and Skewness Parameters of the Log-Stable Distribution for Local-Currency Monthly Returns on 33 Morningstar Country Market Indexes, Jul 1998–Mar 2020



Source: Morningstar Direct.

To see how well the log-stable model works as a model of stock market index returns of countries around the world, I fitted it to the monthly returns of Morningstar stock market indexes on 33 countries (including the U.S. and Canada) over the period of July 1998 to March 2020. In Exhibit 7, I show the results as a scatter plot of the skewness and peakedness parameters. For all countries, the peakedness parameter is less than 2, indicating that the returns on all of the countries have fat-tailed distributions. As for skewness, for all countries other than Korea, the skewness parameter is negative. For eight countries (France, Japan, Italy, South Africa, United States, Mexico, Norway, and Poland), the skewness parameter is at its most negative possible value, negative 1. All of the remaining countries fall in wide range of values between 0 and negative 1.

This shows that country stock market indexes from around the world have historical return distributions that are fat-tailed and left-skewed beyond what standard models predict. Perhaps it is time to reconsider Mandelbrot's work on the distribution of price changes.[11]

A Gaggle of Economists

The ups and downs of the capital markets are often associated with the ups and downs of the economy. This is often called the business cycle. Economists have developed various explanations for the business cycle, perhaps too many explanations. Given how much economists always disagree with each other, it is no wonder that in the 1930s, someone said, "If all economists were laid end to end, they would not reach a conclusion."[12]

11 Markowitz has a different point of view. See Markowitz, Savage, and Kaplan (2010).

12 This quote is attributed to George Bernard Show in May 1933, but there is evidence it was already in circulation by July 1932. See https://quoteinvestigator.com/2016/09/13/economists/.

This is even more true today than it was then, especially regarding competing theories of the business cycle. I refer to the proponents of the competing theories as a gaggle of economists. The reason that there are so many theories of the business cycle is that economist are divided into camps, called schools of economics, each with its own theory of the business cycle. These schools include (but are not limited to) the Keynesian, Monetarist, New Classical, Austrian, and Post-Keynesian schools. Here is a brief description of each school's business cycle theory:

Keynesian

According to the Keynesian school, economic fluctuations are caused by fluctuations in aggregate demand, which includes household consumption, business investment, and government spending. According to Keynesians, when aggregate demand falls short and the economy goes into a recession, the government should step in with increased spending to the fill the gap. While John Maynard Keynes started the Keynesian school with the publication of his book, often called *The General Theory* (Keynes 1936), it was Nobel laureate Paul Samuelson who popularized Keynes' ideas with his university textbook, *Economics*, first published in 1948 (Samuelson 1948). Its 19th edition was published in 2009 (Samuelson and Nordhaus 2009).[13]

Monetarist

Nobel laureate Milton Friedman developed the Monetarist school almost as a direct counter to Keynesianism. According to Friedman, monetary policy (changes to the money supply) is more effective than Keynesian fiscal policy (increasing government spending during a recession). He viewed the business cycle as the result of the mismanagement of monetary policy. In particular, he viewed the cause of the Great Depression as being the large contraction of the money supply by the U.S. Federal Reserve (Friedman and Schwartz 1963).

According to Friedman, the impact of monetary policy depends on the expectations of the agents in the economy. If agents are expecting an increase in the money supply, then increasing the money supply will lead to an increase in prices without stimulating the economy, leaving unemployment unchanged. In his 1967 presidential address to the American Economic Association, emphasizing the role of expectations on the effects of monetary policy, Friedman put forth the possibility of high inflation and high unemployment at the same time, an impossibility according to Keynesian theory (Friedman 1968). Friedman was vindicated during the great stagflation of the 1970s.

New Classical

The New Classical school picks up where Friedman left off regarding expectations. Friedman treats expectations as being adaptive, that is, agents start with expectations based on the past and modify them as events unfold. Based on the work of John Muth (1961), Nobel laureate Robert Lucas changed Friedman's assumption of adaptive expectations to rational expectations. Under the assumption of rational expectations, agents use the correct model of the economy to form their expectations, based on the information that they have. However, the information that agents receive could be a blend of

13 In my freshman year of college, I studied economics with the 10th edition.

local and economywide effects. For example, if a producer sees the price for a good rise, it could be any combination of a local increase in demand for the good and an unexpected increase in the economy-wide money supply. The producer reacts to the price increase based on the model of the economy and the ambiguous information conveyed by the price increase. In this way, unexpected changes in the money supply can have an impact on the real economy, at least temporally. According the New Classical school, only unexpected changes in fiscal or monetary policy cause the business cycle.

Austrian

The Austrian school of economics originated in Vienna in the 19th century. In the 20th century, two of its best-known proponents were Ludwig von Mises and Nobel laureate Friedrich Hayek. The Austrians developed their own theory of the business cycle. Like the Monetarists, the Austrians blame the business cycle on the intervention of the central bank on the economy, but for the opposite reason. The Austrians believe that a monetary expansion in a fractional banking system causes an overexpansion of credit, leading businesses to overinvest and misallocate resources. Eventually, there is a correction, which leads to a recession. Once the correction runs its course, things go back to normal, until the next monetary overexpansion.

Post-Keynesian

The Post-Keynesian school seeks to remain close to Keynes' *General Theory*, yet it has produced some innovations, notably Hyman Minsky's "financial instability hypothesis" (1986, 1992). Minsky argued that market stability is self-destructing because market participants come to believe that markets will remain stable. This causes them to underestimate risk. Their optimism sets things up for an inevitable crisis. Once the crisis is resolved, the markets become calm again, which starts the process again that leads to the next crisis.

Of all the business cycle theories that I have summarized, Minsky's theory is only one that deals directly with the capital markets. It fits well with the facts of the 2007–09 global financial crisis and with the presence of black turkeys and fat tails in global stock market historical return data.

Risks With Rewards

The presence of black turkeys and fat tails in historical stock market return data is undeniable. What is happening today because of the coronavirus is only the most recent example. Yet, they do not exist in the standard statistical models of finance. As for explanations of them and the related phenomenon of business cycles, we have a gaggle of economists offering various conflicting explanations. Such is the state of knowledge in our industry.

Nevertheless, the lessons for investors remain regardless of which school of economics one might subscribe to — market risk is not just about volatility. It is also about the possibility of depressed markets and extreme events. Fortunately, for investors who can stay in the market over the long run, the markets continue to provide rewards for these risks.

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References

Cootner, P. H. 1964. *The Random Character of Stock Market Prices*. (Cambridge, MA: MIT Press).

Friedman, M. 968. "The Role of Monetary Policy." *American Economic Review*, Vol. 58.

Friedman, M. & Schwartz, A.J. 1963. *A Monetary History of the United States*, *1867–1960*. (Princeton, NJ: Princeton University Press).

Goetzmann, W.N., Roger G. Ibbotson, R.G., & Peng, L. 2000. "A New Historical Database for the NYSE 1815 to 1925: Performance and Predictability." *Journal of Financial Markets*, Vol. 4.

Ibbotson, R.G. 2020. *2020 SBBI® Yearbook: Stocks, Bonds, Bills, and Inflation®*. (Chicago: Duff & Phelps).

Kaplan, P.D. 2009a. "Once-and-a-Quarter Centuries of Stock Market Drawdowns." *Morningstar Alternatives Investments Observer*, Third Quarter. Chapter 16 in Kaplan (2012).

Kaplan, P.D. 2009b. "Déjà Vu All Over Again." *Morningstar Advisor*, February/March. Chapter 18 in Kaplan (2012).

Kaplan, P.D. 2009c. "Déjà Vu Around the World." Morningstar Institutional Perspective, October.

Kaplan, P.D. 2010. "Stock Market Bubbles and Crashes: A Global Historical and Economic Perspective." *Morningstar Alternatives Investments Observer*, First Quarter. Chapter 17 in Kaplan (2012).

Kaplan, P.D. 2012. *Frontiers of Modern Asset Allocation*. (Hoboken, NJ: John Wiley & Sons).

Kaplan, P.D. 2015. "The Tails That Wag the Dog." *Morningstar*, August/September.

Kaplan, P.D. 2018. "Filling in the Gaps." *Morningstar*, August/September.

Kaplan, P.D. 2019. "Further Beyond the Bell Curve." *Morningstar*, Spring.

Kaplan, P.D., Cooper, G., Ibbotson, R., & Mandelbrot, B. 2009. "Getting a Read on Risk: A Discussion With Roger Ibbotson, George Cooper, and Benoît Mandelbrot." *Morningstar Advisor*, February/March. Chapter 20 in Kaplan (2012).

Kaplan, P.D., Idzorek, T., Gambera, M., Yamaguchi, K., Xiong, J., & Blanchett, D. 2009. "The History and Economics of Stock Market Crashes." In Siegel (2009).

Keynes, J.M. 1936. *The General Theory of Employment, Interest, and Money*. (New York: Harcourt Brace and Company).

Lintner, J. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *Review of Economics and Statistics*, February.

Malkiel, B.G. 2019. *A Random Walk Down Wall Street*. (New York: W.W. Norton & Company).

Markowitz, H.M. 1952. "Portfolio Selection." *Journal of Finance*, Vol. 7.

Markowitz. H.M. 1959. *Portfolio Selection: Efficient Diversification of Investments*. (New York: J. Wiley).

Minsky, H.P. 1986. *Stabilizing an Unstable Economy*. (New Haven, CT: Yale University Press).

Minsky, H.P. 1992. "The Financial Instability Hypothesis." The Jerome Levy Economics Institute of Bard College, Working Paper 74 (May). www.levy.org/pubs/wp74.pdf.

Morningstar, Inc. 2011. "Morningstar Asset Allocation Optimization Methodology." Morningstar Methodology Paper, September.

Muth, J.F. 1961. "Rational Expectations and the Theory of Price Movements." *Econometrica*, Vol. 29, No. 3.

Pierce, P.S. 1982. *The Dow Jones Averages, 1885–1980*. (Homewood, IL: Dow Jones Irwin).

Ross, S.A. 1976. "The Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory*, Vol. 13, No. 3.

Samuelson, P.A. 1948. *Economics*. (McGraw-Hill Book Company).

Samuelson, P.A. & Nordhaus, W.D. 2009. *Economics*. (McGraw-Hill Book Education).

Satell, G. 2010. "Remembering Benoît Mandelbrot." *Kyiv Post*, Oct. 19, 2010.

Sharpe, W.F. 1964. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance,* Vol. 19, No. 3.

Siegel, L.B. 2009. *Insights Into the Global Financial Crisis*. (Charlottesville, VA: CFA Institute).

Siegel, L.B. 2010. "Black Swan or Black Turkey? The State of Economic Knowledge and the Crash of 2007–2009." *Financial Analysts Journal*, July/August.

Taleb, N.N. 2007. *The Black Swan: The Impact of the Highly Improbable*. (New York: Random House).

Xiong, J.X. 2010. "Nailing Down Risk." *Morningstar Advisor*, February/March.

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